Assignment 5

MATH-F310: Differential Geometry I

October 26, 2022

1. Prove that the set of critical points of a smooth function on a surface is closed and hence compact if the surface is compact.

2. The holomorphic map $z \mapsto z^2$ extends to a smooth map $f : S^2 \to S^2$ using stereographic projection (see assignment 3, question no. 2). Compute the differential of this map Df_p at a point $p \in S^2$. Find out the critical points of f.

3. Prove that the vector space of smooth functions on a surface is infinite dimensional.

4. Let $S \subset \mathbb{R}^3$ be a surface. For any smooth function $f : S \to \mathbb{R}$ and any $p \in S$, $\exists$ an open neighbourhood $W \subset \mathbb{R}^3$ around p, and a smooth function $h : W \to \mathbb{R}^3$, such that $h|_{S\cap W} = f|W$.

5. Let S be a surface in $\mathbb{R}^3$. Let $p_0 \in \mathbb{R}^3 \setminus S$. Show that the shortest line segment from p_0 to S (if one exists) is perpendicular to S; i.e., show that if $p \in S$ is such that $||p_0 - p||^2 \leq ||p_0 - q||^2$ for all $q \in S$, then the line segment $p_0 - p$ at p is perpendicular to T_pS.