Exercise sheet 6

Algebraic Topology

November 19, 2024

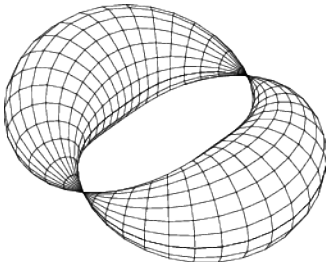

Figure 1: *The double pinched torus*

Exercise 1. Show that the homology groups of the double pinched torus in Figure 1 are

$$H_k(K) \simeq \begin{cases} \mathbb{Z} & \text{for } k = 0, 1 \\ \mathbb{Z}^2 & \text{for } k = 2 \\ 0 & \text{else.} \end{cases}$$

Exercise 2. Show that the homology groups of the connected sum between the torus and $\mathbb{R}P^2$ are

$$H_k(K) \simeq \begin{cases} \mathbb{Z} & \text{for } k = 0 \\ \mathbb{Z}^2 \oplus \mathbb{Z}_2 & \text{for } k = 1 \\ 0 & \text{else.} \end{cases}$$

Exercise 3. Using Mayer-Vietoris, show that the homology groups of $\mathbb{R}P^3$ are

$$H_k(K) \simeq \begin{cases} \mathbb{Z} & \text{for } k = 0, 3 \\ \mathbb{Z}_2 & \text{for } k = 1 \\ 0 & \text{else.} \end{cases}$$

(Hint: Recall that $\mathbb{R}P^3$ is the quotient of S^3 under a $\mathbb{Z}_2$-action. Can you find a covering of S^3 that is $\mathbb{Z}_2$-invariant? How does this covering look like in the quotient?)