# Exercise sheet 7

Algebraic Topology

November 26, 2024

**Exercise 1.** Let $X$ be a CW complex and let $C$ be a compact subset of $X$. Show that $C$ only intersects a finite number of cells.

**Exercise 2.** Show that the pair $(\mathbb{R}^n, S^{n-1})$ has the homotopy extension property.

**Exercise 3.** Show that the unit sphere inside an infinite dimensional Banach space cannot be viewed as a CW complex. *Hint: Use that a Banach space is a Baire space, i.e. every countable intersection of dense open sets is dense.*

**Exercise 4.** Compute the homology of the following 2-complexes:

1. $S^1 \times (S^1 \vee S^1)$
2. The space obtained from a closed disk $D^2$ by first deleting the interiors of two disjoints subdisks in the interior of $D^2$ and then identifying all the three resulting boundary circles via homeomorphisms that preserve the clockwise orientations of these circles.